# Quantum Fluid Turbulence

## Predicting Bubble Fragmentation in Superfluids

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## 1 Introduction

Quantum theory transformed our understanding of the subatomic world by showing that particles in bound systems have distinct energy levels. In this realm, objects exhibit both particle and wave-like behaviours, and their interactions are guided by probabilities [1]. One fascinating outcome of this is the **Bose-Einstein condensate**, where particles can be cooled to extremely low temperatures, causing their wave properties to overlap. This results in a peculiar "blur" of particles behaving like a massive wave of matter.

Bose-Einstein condensates (BECs) are superfluids and have no viscosity, meaning they flow without being slowed by friction. Gently moving a solid object through a superfluid doesn't disturb this flow, however, exceeding a critical velocity breaks the superfluidity, creating empty regions known as vortices and introducing turbulence.

**The aim of this project is to move a secondary unmixable fluid through a primary fluid in order to achieve the following objectives:**

**$\alpha$ Theoretical analysis of the dynamics in a binary superfluid system.**

**$\beta$ Identification and description of new regimes in turbulent superfluid flow.**

**$\gamma$ Prediction of these regimes based upon initial conditions.**

This project explores two-component 2D superfluid turbulence, an idealised viscosity-free model of the chaotic turbulence found throughout nature, from the blood rushing through our veins to astrophysical events. Its significance lies in:

- Extending to 3D surfaces - mapping atmospheric turbulence onto spheres.
- Experimental applications - imaging real BECs with secondary components.

## 2 Methods

Bose-Einstein condensate superfluids are described by a wavefunction ($\psi$), which adheres to a non-linear wave equation known as the **Gross-Pitaevskii equation**. This project's two 2D fluids are governed by a coupled variant of this equation,

$$\frac{\partial\psi_1}{\partial t} = -i\left[-\frac{1}{2}\nabla^2 + |\psi_1|^2 + g_{12}|\psi_2|^2 - 1\right]\psi_1,$$
$$\frac{\partial\psi_2}{\partial t} = -i\left[-\frac{1}{2}m\nabla^2 + g_{12}|\psi_1|^2 + g_{22}|\psi_2|^2 - \mu\right]\psi_2,$$

where $g_{12}$ accounts for interactions between fluids, while $g_{22}$ describes those within one fluid. The ground states of the system were identified for varying numbers of secondary fluid particles, and the secondary fluid in these states were moved, as illustrated in Figure 1. The time evolution of the system was then simulated.

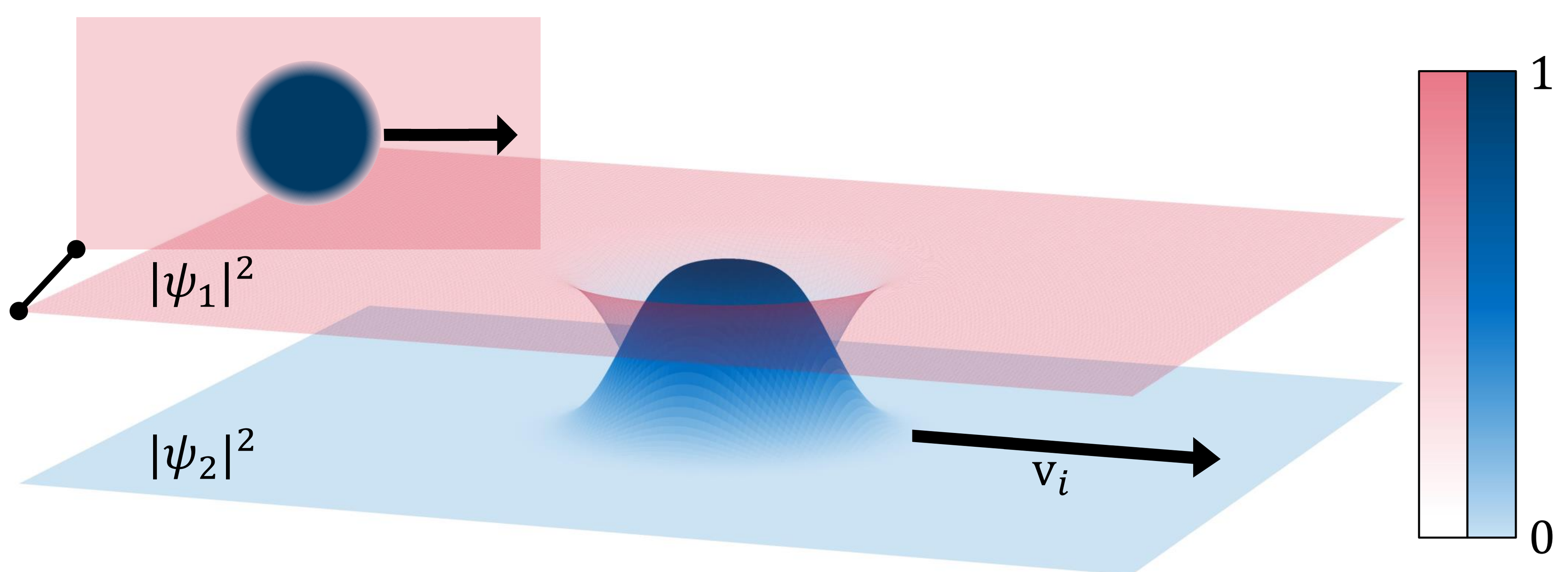


**Figure 1.** The initial ground state densities of the primary component ($|\psi_1|^2$) and the secondary component ($|\psi_2|^2$). The secondary component "bubble" is boosted with a given velocity ($v_i$), after which it will interact with the primary component.

## 3 Results

$\alpha$ Simulations were successfully run for different initial "bubble" conditions, altering the number of fluid particles and their initial velocities. Vortices were observed to form in phase-linked (special synchronised spinning) pairs with opposite circulation, called **vortex-antivortex pairs**. Figure 2 displays the number of pairs observed.

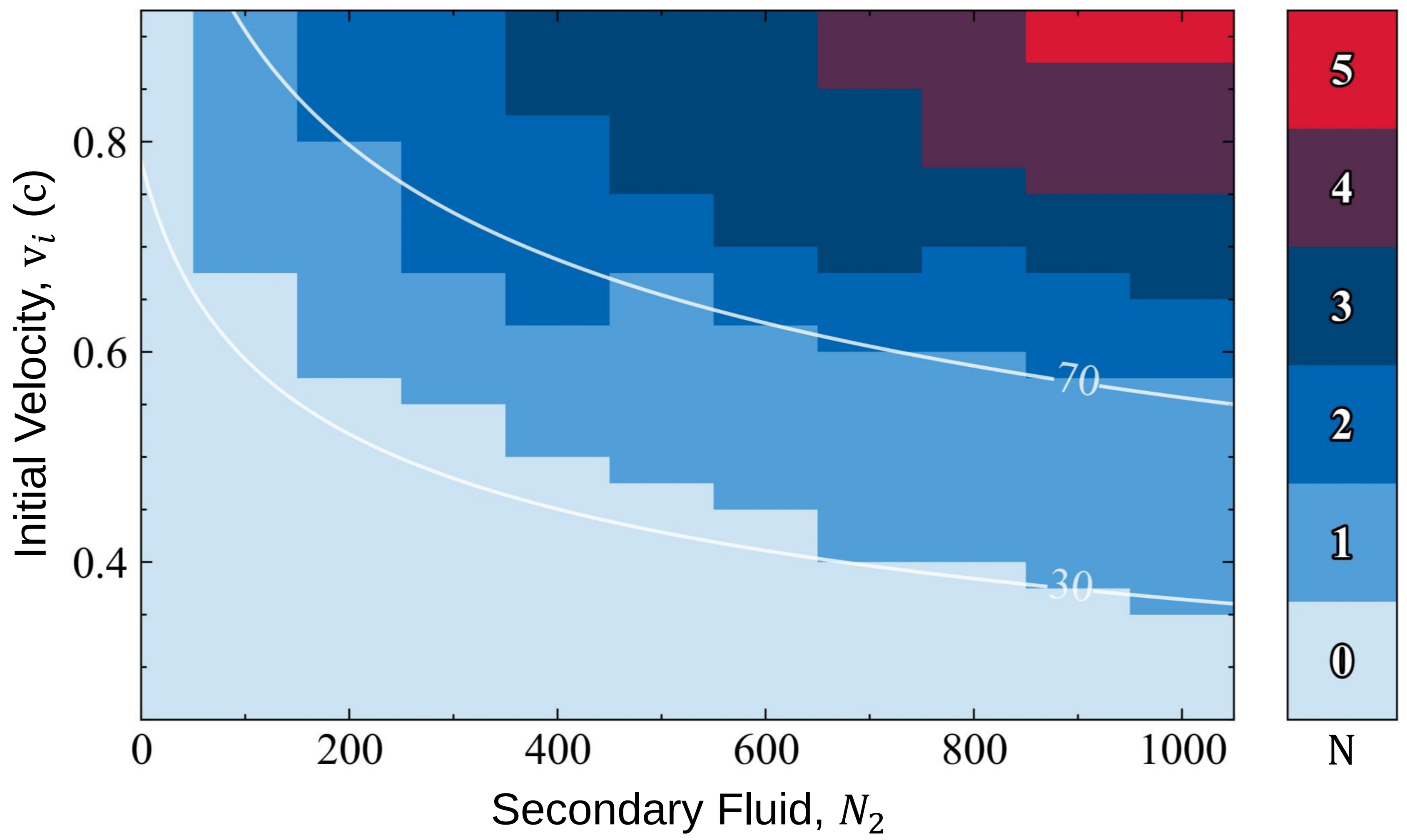


**Figure 2.** The number of vortex-antivortex pairs formed (N) across the number of secondary fluid particles ($N_2$) and initial velocity ($v_i$) parameter space. The contours are given by a quantum analogue to the Weber number.

$\beta$ From the observed vortex dynamics, distinct patterns and regimes of turbulence were identified, as illustrated in Figure 3.

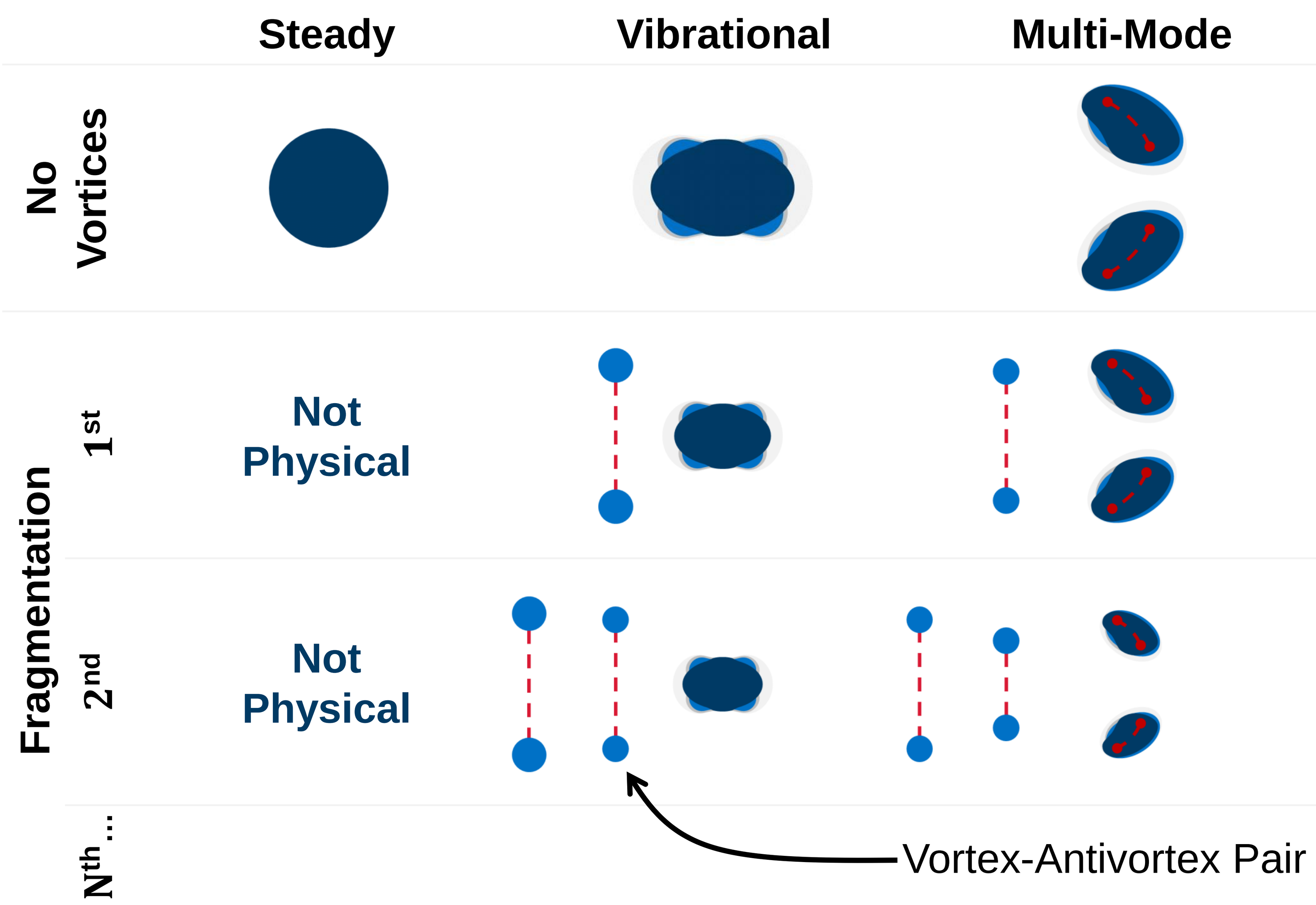


**Figure 3.** Characteristic regimes of superfluid turbulence, where N is the number of vortex-antivortex pairs formed. The darker blue represents regions of significant density in the secondary component, while the vortex-antivortex pairs are depicted in lighter blue. Formed or forming phase-links are indicated by red dashed lines.

## 4 Discussion

$\alpha$ After boosting past a critical velocity, the immiscibility of the two fluids meant that the bubble acted as an obstacle, shedding vortices and introducing turbulence. This was expected from well-established prior research into solid obstacles [2]. However, with the obstacle being fluid, its inherent ability to deform gave rise to the fascinating phenomenon of the secondary fluid infilling the vortex cores.

$\beta$ The identified regimes closely align with classical liquid breakup dynamics. The dynamics shift from a vortex-free steady state to a vibrational state before entering a multi-mode state as velocity increases. Surpassing the critical velocity initiates the first fragmentation regime, starting with vibration and evolving into a multi-mode state. These regimes repeat with sufficiently large bubbles.

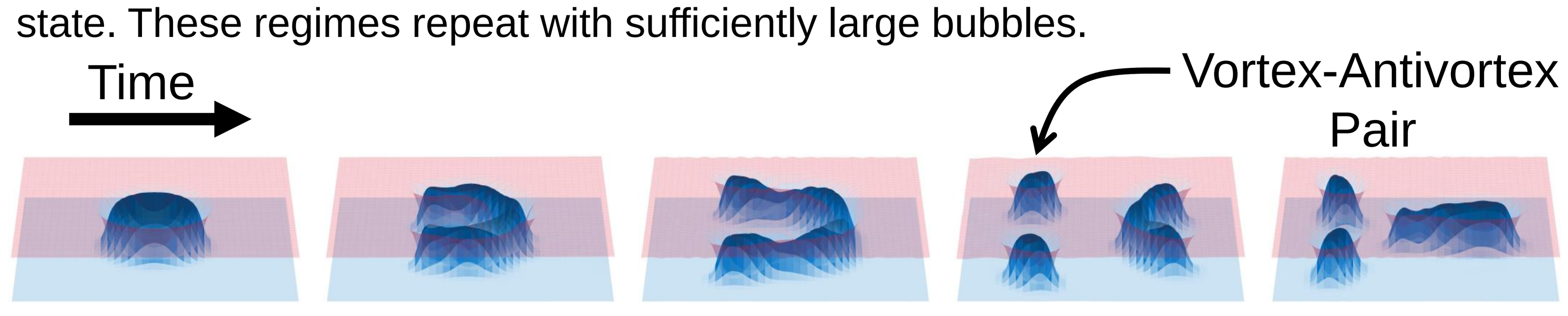


**Figure 4.** Sample dynamics for $N_2 = 1000$ at $v_i = 0.45$ c, beyond the critical velocity in the 1st fragmentation vibrational scheme. The vortex cores are clearly infilled.

$\gamma$ The **Weber number** characterises the classical flow of multi-fluid interactions,

$$\mathrm{We} = \frac{\rho \ell v^2}{\sigma},$$

where $\rho$ is the density, $\ell$ is the characteristic length, v is the velocity and $\sigma$ is the surface tension. Classification of the simulated dynamics can be predicted by a quantum analogue to the Weber number, with 1st fragmentation at $\mathrm{We} \sim 30$.

## 5 Conclusion

**This project achieved its objectives as follows:**

**$\alpha$ Successfully simulated dynamics in a binary superfluid system, resulting in infilled vortex-antivortex pairs after reaching a critical velocity.**

**$\beta$ Introduced new characteristic regimes, providing a comprehensive description of turbulent superfluid flow.**

**$\gamma$ Obtained promising results for predicting these regimes using a quantum Weber number.**

Ultimately, turbulence remains a challenging puzzle, but every advance in research brings us closer to the significant benefits it promises. As Richard P. Feynman aptly stated, "*Turbulence is the most important unsolved problem of classical physics*."

## Acknowledgements

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## References

[1] C. F. Barenghi and N. G. Parker, "*A Primer on Quantum Fluids*", Springer, Berlin (2016).

[2] M. T. Reeves, T. P. Billam, et al. "*Identifying a Superfluid Reynolds Number via Dynamical Similarity*", Phys. Rev. Lett. **114** (2015).