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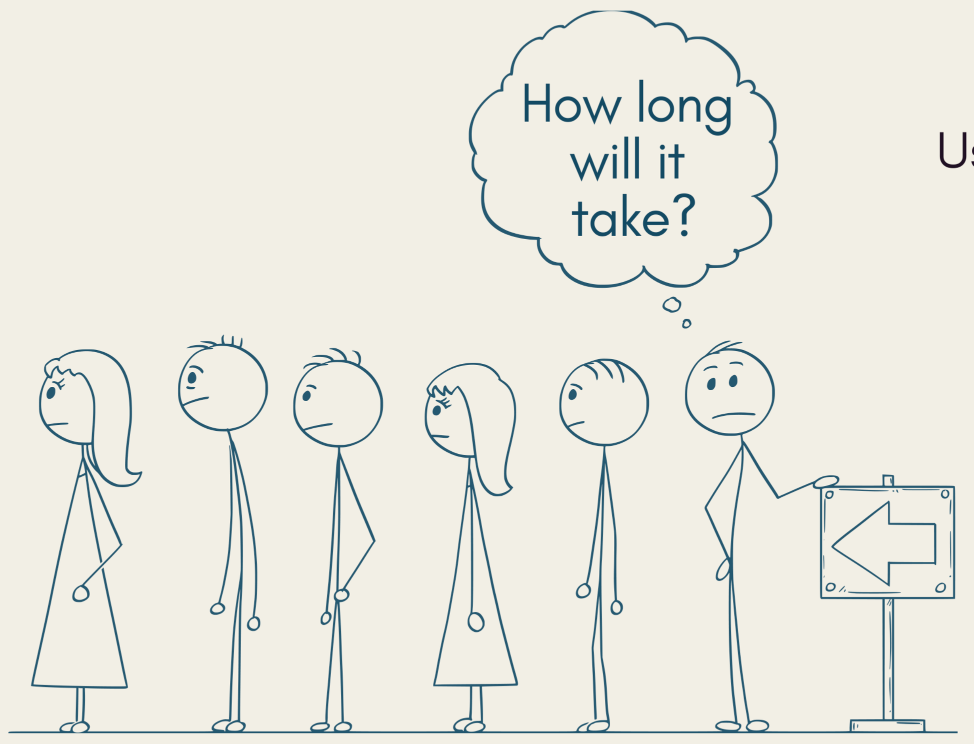
MATHEMATICAL QUEUEING MODELS AND THEIR APPLICATIONS IN EVERYDAY PROBLEMS

Have you ever thought about how much time you spend waiting in lines? On average, people spend around **five years** of their lives waiting in queues.

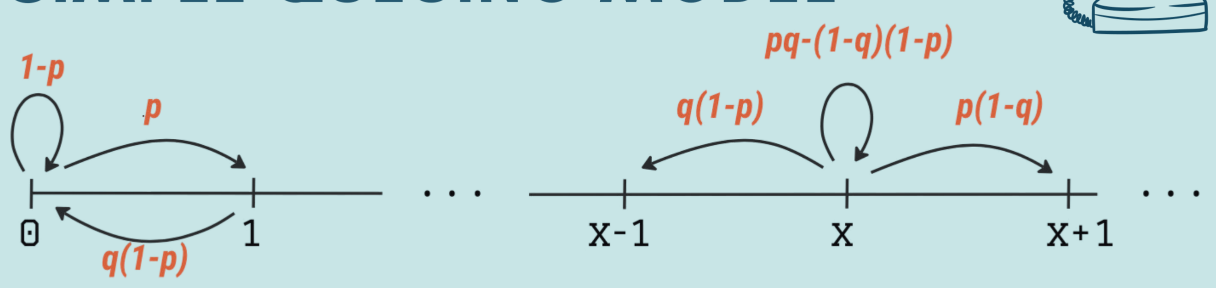
Join us on this analytical journey where theory meets code and modelling reveals the operational principles driving traffic dynamics.



Using probability theory and stochastic processes, we gain insight into the dynamics of traffic flows.

Our Python simulation models recreate real traffic scenarios, allowing us to uncover their complexity. Markov chains track transitions and Monte Carlo simulations generate scenarios, revealing underlying patterns.

SIMPLE QUEUING MODEL



X_n is the number of customers waiting in line.
This is a Markov chain with state space {0, 1, 2, ...} and transition probabilities:

$$p(x, x-1) = q(1-p) \quad p(x, x) = qp + (1-q)(1-p)$$

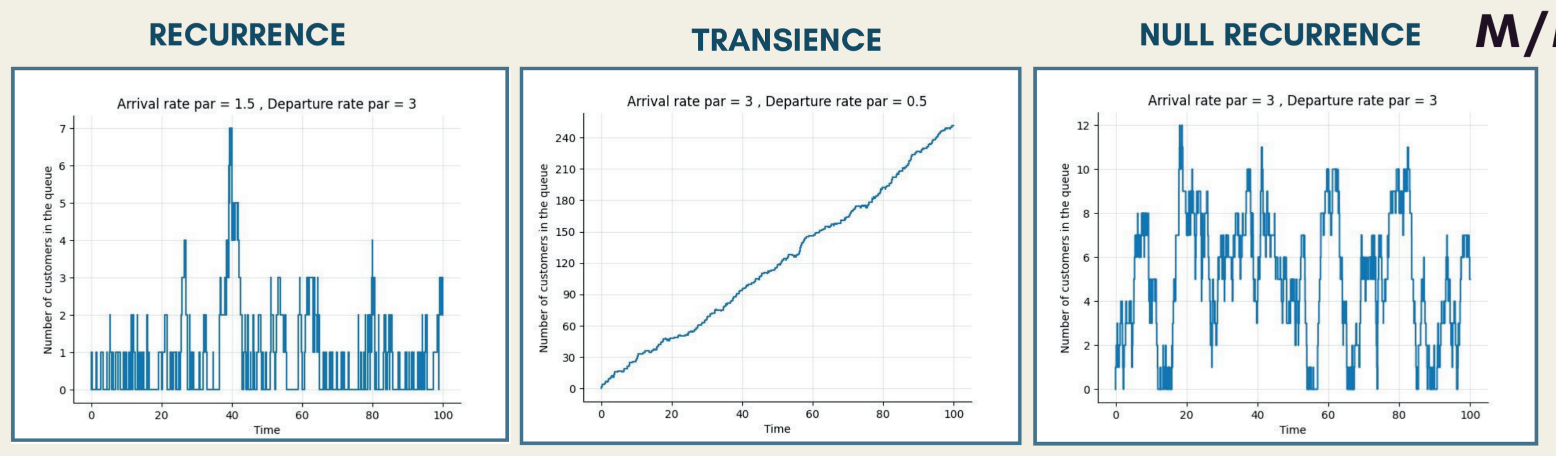
$$p(x, x-1) = q(1-p) \qquad x > 0$$

$$p(0,0) = 1-p \qquad p(0,1) = p$$

QUEUE MODELLING & BIRTH-AND-DEATH CHAINS

X_t denotes the number of people in line for some service.
The arrival of customers follows a Poisson process with rate λ.
Customers are serviced at an exponential rate μ.

Simulation examples: queue M/M/1



M/M/1 QUEUE - ONE SERVER

Arrival rate = λ
Departure rate = μ

M/M/K QUEUE - K SERVERS

Arrival rate = λ

Departure rate = $\begin{cases} n\mu & \text{if } n \le k \\ k\mu & \text{if } n \ge k \end{cases}$

POISSON PROCESS

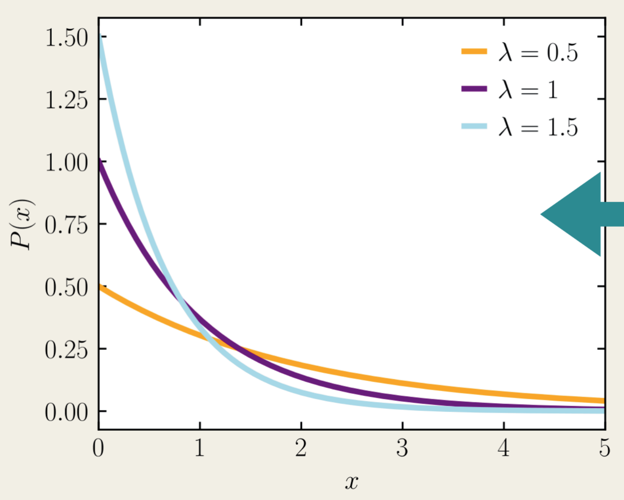
Consider X_t the number of customers arriving in a line by time $t,\ t \in \mathbb{R}_0^+$

1. The number of customers arriving during a one-time interval does not affect the number arriving during a different time interval.
2. The "average" rate at which customers arrive remains constant.
3. Customers arrive one at a time.

$$X_t \sim Poisson(\lambda t)$$

A stochastic process X_t with $X_0 = 0$ satisfying these assumptions is called a Poisson process with rate parameter λ.

$T_n \sim Exp(\lambda)$
waiting time between customers follows an exponential distribution



</> **Algorithm concept**: ask me

```
while current time < simulation time: #simulate
    arrival time = exponential random variable (λ)
    departure time = exponential random variable (μ)

  if arrival time < departure time: number of customers += 1
  else:
      if number of customers > 0: number of customers -= 1
```
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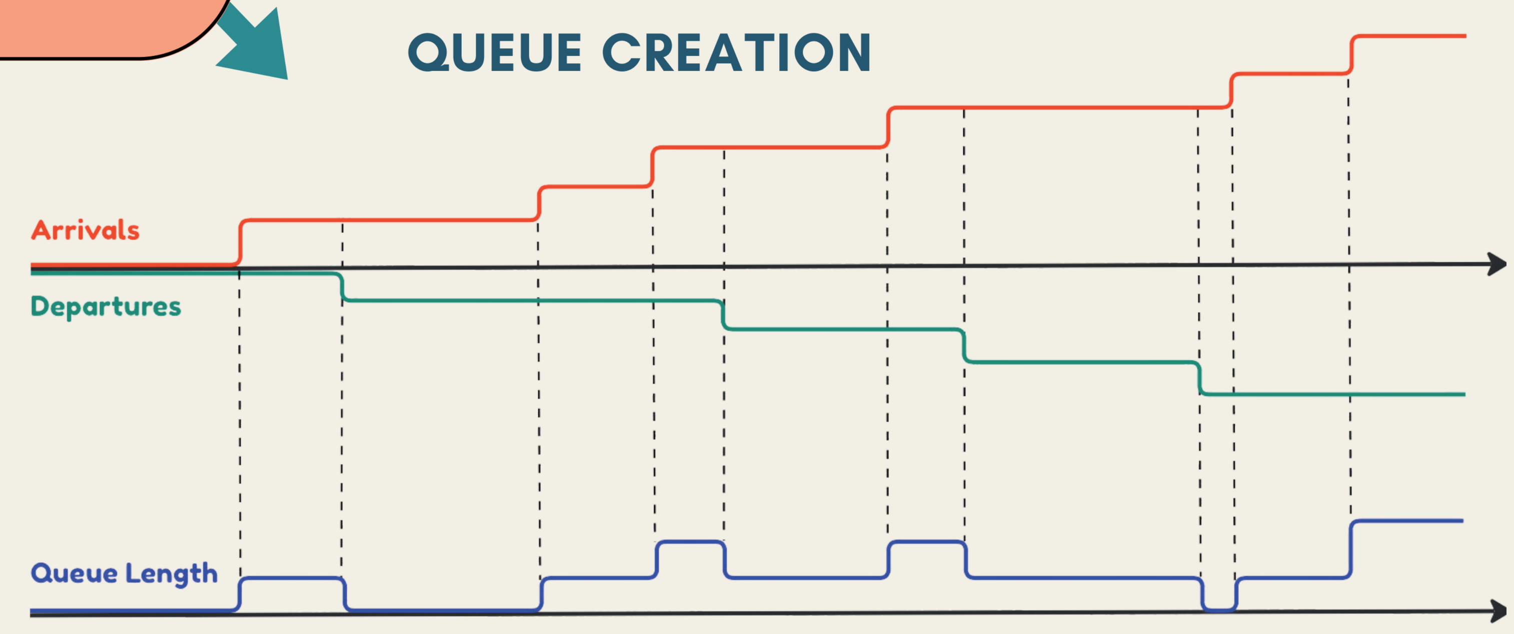
QUEUE CREATION



TRAFFIC HYDRODYNAMIC LIMIT

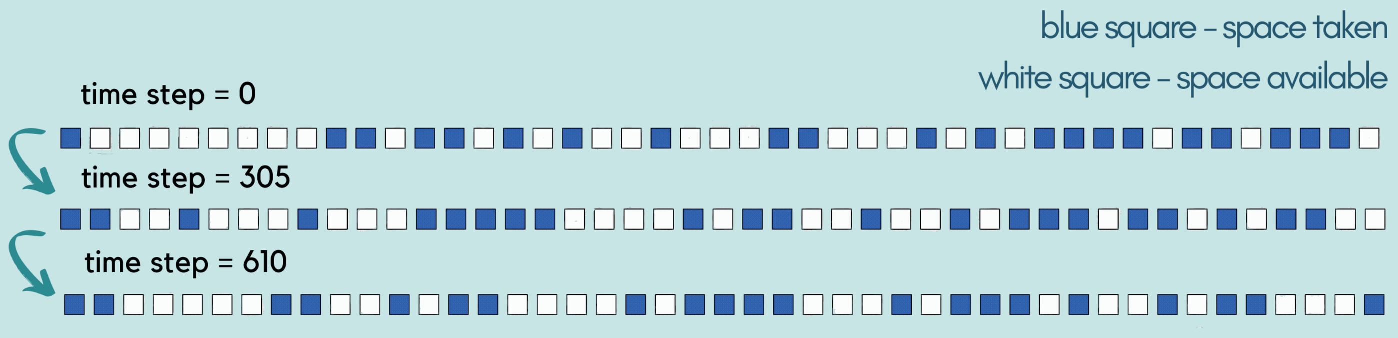
Consider the one-dimensional flow of vehicles.

- x - distance variable
- t - time variable
- c(x) - velocity
- u(x,t) - car density at point x and time t

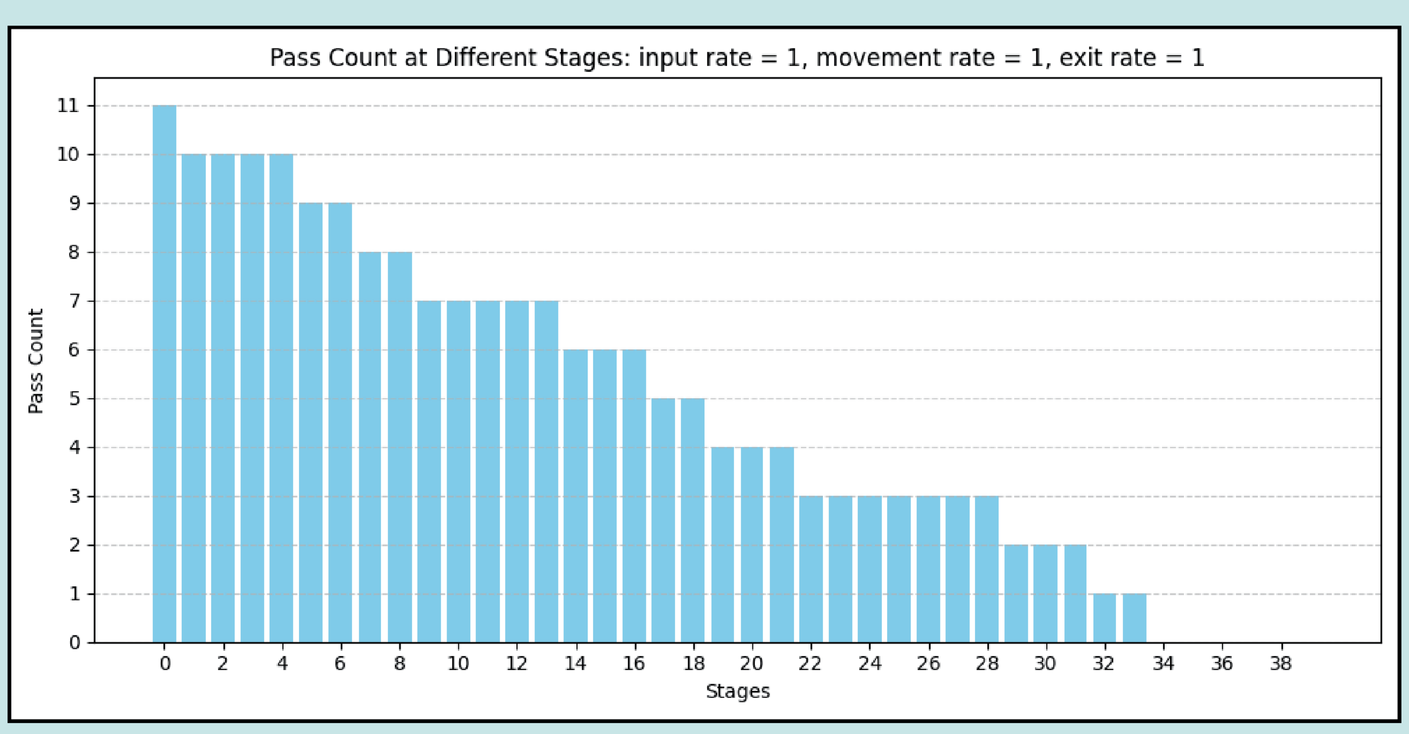
$$\frac{\partial u}{\partial t} + \frac{\partial (cu(1-u))}{\partial x} = 0$$

TRAFFIC MODELLING

EXAMPLE № 1: TRAFFIC FLOW
Input par. = 1, movement par. = 1, output par. =1



HEIGHT FUNCTION



SCAN ME

Scan the QR code to animate the traffic, view the full research report and references

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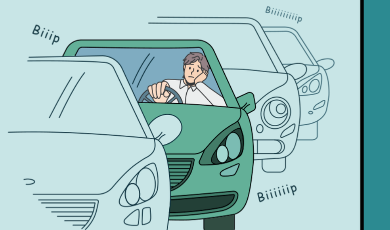
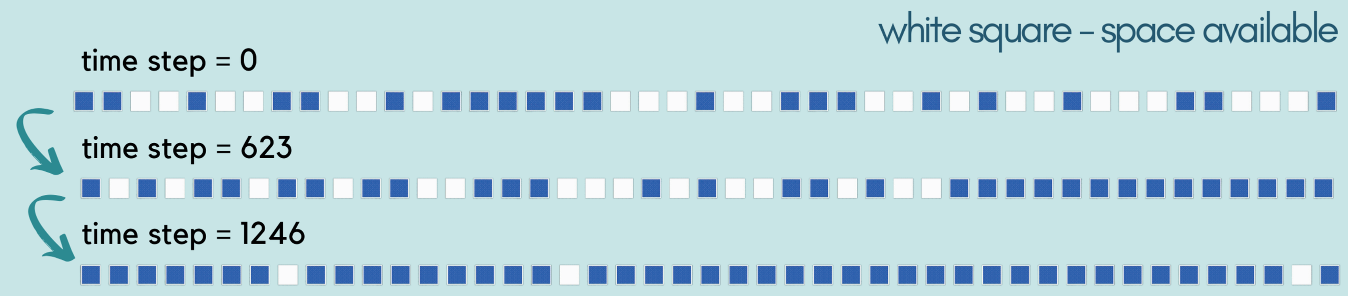


EXAMPLE № 2: TRAFFIC JAM FORMATION
Input par. = 15, movement par. = 10, output par. =1



HEIGHT FUNCTION